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TO: Technical Reports Officer
        Office of Grants and Research Contracts
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        Washington, D. C.   20546

The work on this Grant started on February 1st, 1966. The
Principal Investigator has participated in this work to the ex-
tent of 25% of his time until May 31st, 1966, and 50% of his time
for the months of June and July of 1966. He was joined on June
1st by a graduate student, Ph.D. candidate, who worked on this
Grant full-time in the summer, and, beginning with September 1st,
1966, a second graduate student will be working on this Grant.

To accomplish the goals stated in the proposal, the first
effort was centered on the application of the proposed method to
a simple system, such as an annular membrane, subjected to non-
symmetric normal pressure. The differential equation which gov-
erns the deflection of the membrane is

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} = p \tag{1}$$

where w is the deflection, p is the pressure and r is the radial
and θ the circumferential coordinate. Boundary conditions were
assumed to be $w(a) = w(b) = 0$, and the pressure was taken as
$p = \cos 2\theta$. The object of this investigation was to find out
how many pivotal points around the circumference are needed for
the new method to give accurate results.

This boundary value problem was solved in two ways: (1) separating variables, i.e., assuming $w(r,\theta) = w_2(r) \cos 2\theta$, and (2) replacing $\partial^2 w/\partial^2 \theta$ by the first central difference formula in terms of the pivotal values of w around the circumference. For the first solution, the multisegment method of integration[1] was used while for the second solution, it was calculated with the method which is described in the proposal. Since the solution is symmetric about the line $\theta = 0$, 11 pivotal points around the half-circle were tried. The number of fundamental variables of Eq. (1) is two, which means that a system of 22 first-order linear ordinary differential equations with 22 unknowns had to be solved. If the membrane itself is axially symmetric, then two such initial value problems must be solved.

As seen from the results shown in Figure 1, it can be concluded that if the membrane is axially symmetric and if the load variation is cos 2θ, then 11 points around half-circle give an accuracy of 4 significant digits. These results were regarded as very encouraging, especially since the program ran for 4.5 minutes on a GE 225 computer which has about one-eighth of core storage and one-hundredth of the speed of an IBM 7094.

The next question to be answered was the following: does the new method work as well for systems of first-order nonlinear ordinary differential equations? In order to answer this question, a computer program was written which can calculate the non-symmetric deformation of a circular plate by means of von Karman

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[1] A. Kalnins, Journal of Applied Mechanics, Vol. 31, September 1964, pp. 467-476.

FIGURE 1

DEFLECTION OF AN ANNULAR MEMBRANE UNDER UNIFORM PRESSURE

Point	w Separable	New Method	β Separable	New Method
1	0.12445 E-02	0.12445 E-02	-0.43278 E-04	-0.43298 E-04
3	0.38458 E-03	0.38464 E-03	-0.13374 E-04	-0.13379 E-04
5	-0.10068 E-02	-0.10069 E-02	0.35013 E-04	0.35028 E-04
7	-0.10068 E-02	-0.10070 E-02	0.35013 E-04	0.35028 E-04
9	0.38458 E-02	0.38468 E-03	-0.13374 E-04	-0.13379 E-04
11	0.12445 E-02	0.124472 E-02	-0.43278 E-04	-0.43297 E-04

$p = \cos 2\theta$
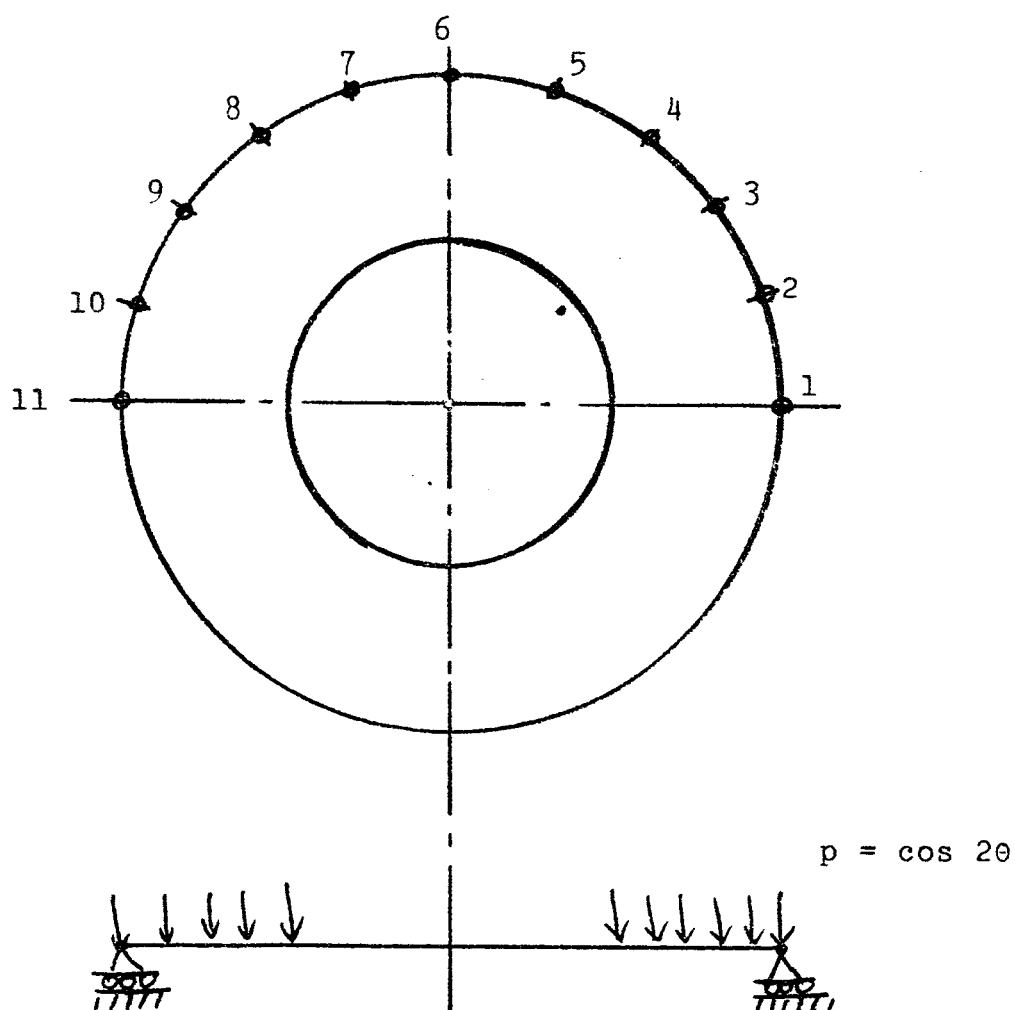
large deflection plate theory. To account for the nonlinearity, the recently developed nonlinear multisegment method was employed[2]. For comparison, the same problem was also solved for the linear case by means of the linear multisegment method. Of course, no comparison is possible for the nonlinear solution because this problem has not been solved before. The corresponding results are shown in Figure 2.

Because of the small size of the computer, only 5 points were taken around the circumference, and since eight fundamental variables are present, then if the plate is axisymmetric, 5 initial value problems involving 40 first-order ordinary differential equations must be solved. The program ran for about six minutes on the GE 225 per iteration. As seen from Figure 2, the comparison of the new method and the exact solution for the linear case shows accuracy of up to 2 significant digits.

The conclusions from these two examples are as follows:

1. If the circumferential variations of system or the loads are not rapid, five to eleven points around the half-circle are sufficient to solve the problem.

2. Even on such a limited computer as a GE 225, there is no difficulty with round-off errors when solving initial value problems of up to 40 first-order ordinary differential equations.

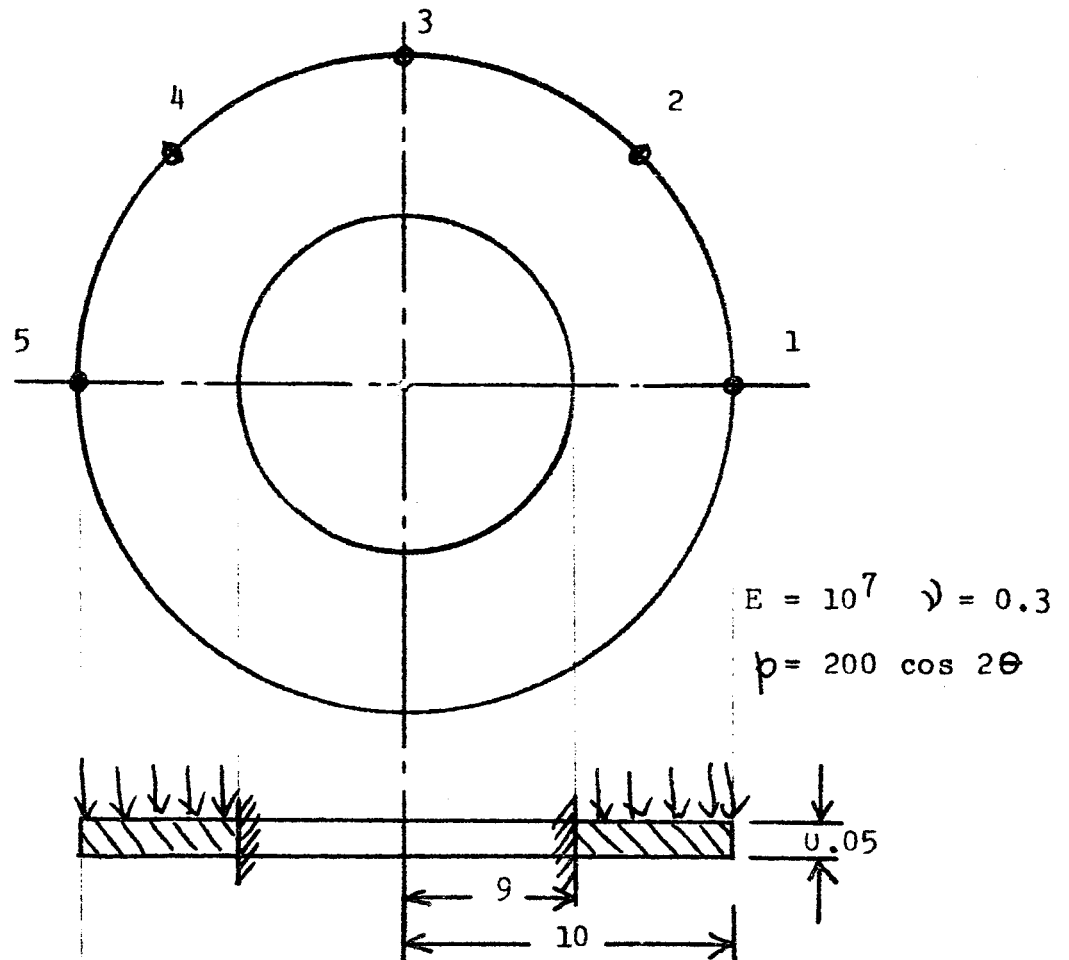3. In the opinion of the Principal Investigator, the method

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[2] A. Kalnins and J. F. Lestingi, "On Nonlinear Analysis of Elastic Shells of Revolution", to be published in Journal of Applied Mechanics.

FIGURE 2

DEFLECTION OF ANNULAR PLATE UNDER UNIFORM
PRESSURE BY VON KARMAN NONLINEAR PLATE THEORY

NORMAL DEFLECTION

Point	Linear		Nonlinear
	Separable	New Method	New Method
1	0.22136 E-00	0.22400 E-00	0.20887 E-00
2	0.0	0.39664 E-02	0.47903 E-02
3	-0.22136 E-00	-0.22600 E-00	-0.21086 E-00
4	0.0	-0.19800 E-02	-0.37897 E-02
5	0.22136 E-00	0.22402 E-00	0.20881 E-00

$E = 10^7 \quad \nu = 0.3$

$p = 200 \cos 2\theta$

described in the proposal can be applied to arbitrary shells of revolution.

A meeting with Dr. R. W. Leonard of Langley Research Center has been arranged for September 30th, 1966, to present these results. The Principal Investigator will propose at this meeting the application of the new method for the analysis of curved shells as described in the enclosed abstract of a future publication.

<center>Abstract of</center>

<center># DEFORMATION OF CURVED, COMPOSITE, THIN-WALLED SHELLS</center>

<center>by</center>

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The purpose of this paper is to develop a method of analysis for a composite shell whose elements can be of two different types: (1) axially symmetric shells of arbitrary shape, and (2) curved shells in the form of a sector of a torus. An example of the type of a shell considered is shown in Figure 1.

The proposed approach to this problem utilizes an extension of the multisegment method of integration [1][1] of the linear shell equations, which permits each element of a composite shell to be analyzed as a separate system. The advantage of this approach is that the elements of the shell can be arranged in any desired order as long as the total shell remains continuous. Such an approach admits great versatility. For example, the curve passing through the centers of the circles which generate the reference surface of the shell need not be a plane curve. By properly rotating the toroidal element, it can become a space curve.

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[1] Numbers in brackets refer to the References listed at the end of this abstract.

Owing to the presence of the toroidal sector of the shell, whose principal radii of curvature of the reference surface are given by $R_\theta = b = $ constant, $R_x = b + a/\sin\theta$ (see Figure 1), the solution in the total shell is no longer separable in the two surface coordinates (as it is in a straight axisymmetric shell), and the problem does not lend itself directly to the one-dimensional analysis as given in Reference [1]. However, the method of Reference [1] can be modified in such a way that the necessity of separability of the solution is eliminated.

The extension of the multisegment method is concerned with the following boundary-value problem: find $y(x,\theta)$, governed in a region ($a \leq x \leq b$, $0 \leq \theta \leq 2\pi$) by the system of partial differential equations

$$\frac{\partial y(x,\theta)}{\partial x} = F\left[x,\ \theta,\ y(x,\theta),\ \frac{\partial y(x,\theta)}{\partial \theta},\ \frac{\partial^2 y(x,\theta)}{\partial \theta^2}\right] \tag{1a}$$

and the following boundary conditions:

$$T_a\ y(a,\theta) = u_a$$
$$T_b\ y(b,\theta) = u_b \tag{1b}$$
$$y(x,0) = y(x,2\pi)$$

Here, y denotes an $(8,1)$ column matrix whose elements are the unknown dependent variables; F represents 8 functions arranged in a column matrix form; T_a, T_b, are $(4,4)$ and u_a, u_b, are $(4,1)$ matrices, respectively, whose elements are specified by the boundary conditions; x and θ are the independent variables.

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Following, for example, the derivation carried out in Reference [2], the boundary value problem of a shell can be stated in the form as given by Eqs. (1). For an axially symmetric reference surface, the principal radii of curvature are functions of the meridional coordinate x only, while in the toroidal sector R_x does vary with θ. The θ-variation of R_x rules out a separable solution in x and θ, and a modification of the method of Reference [1] is necessary.

According to the method proposed in this paper, all dependent variables and the θ-dependent coefficients occuring in Eqs. (1) are expressed by their pivotal values and their derivatives by finite differences in the θ direction, while direct multisegment, numerical integration is performed in the x direction in the same way as in Reference [1]. Such an approach requires initial-value integrations of a system of $8N$ ordinary differential equations, where N denotes the number of pivotal points selected around the circumference of each circle generated by the plane x = constant. In most cases, the variation of the solution in the θ-direction is not rapid, and consequently N need not be more than, say, twelve. Moreover, in the presence of symmetry of the solution with respect to a plane, this number can be halved. With the aid of a high-speed digital computer, the initial-value integration of the differential equations presents no fundamental difficulties. The rest of the analysis parallels that given on pages 473-474 of Reference [1].

The paper includes the derivation of Eqs. (1a) for an axially symmetric shell as well as for a toroidal element. Both sets of equations possess the feature that no derivatives with respect to x or θ of the shell-parameters (radii of curvature, thickness, etc.) are present. This fact makes the calculation of the derivatives of the 8N variables at any value of x much easier.

For the demonstration of the feasibility of this method, the numerical results of two examples are planned to be presented in the paper. One example is the bending of a curved tube treated by a different method in Reference [3], and the other example is the type of a shell shown in Figure 1 when subjected to a load P which is uniformly distributed around the circumference.

In summary, a versatile method is developed for the analysis of composite shells which consist of elements of axially symmetric and curved shells.

References

1. A. Kalnins, "Analysis of Shells of Revolution Subjected to Symmetrical and Nonsymmetrical Loads", Journal of Applied Mechanics, vol. 31, 1964, pp. 467-476.

2. A. Kalnins, "On Free and Forced Vibration of Rotationally Symmetric Layered Shells", Journal of Applied Mechanics, vol. 32, 1965, pp. 941-943.

3. H. Karl, "Biegung gekrümmter, dünnwandiger Rohre", ZAMM, vol. 23, 1943, pp. 331-345.
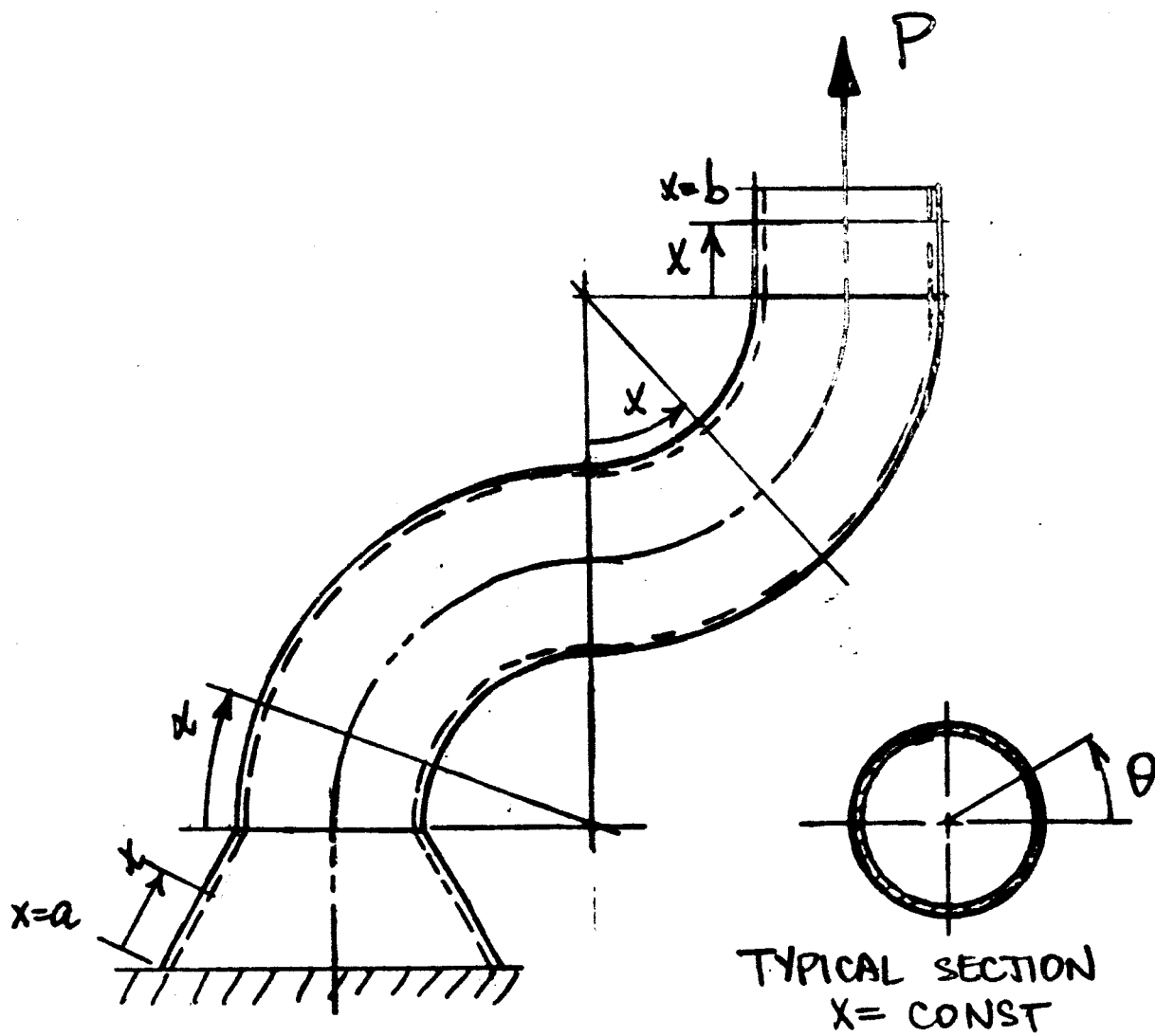
P

x=b

x

x

α

x

x=a

θ

TYPICAL SECTION
X= CONST

FIGURE 1